

**TTIC 31230 Fundamentals of Deep Learning  
Problems For Fundamental Equations.**

**Problem 1. The Zero Temperature (Infinite  $\beta$ ) Limit.**

Suppose we introduce a temperature parameter into the softmax operation.

$$P_{\beta \text{ softmax}}(y) = \frac{1}{Z} e^{\beta s(y)} \quad Z = \sum_y e^{\beta s(y)}$$

suppose that there exists an element  $y_{max}$  such that for all  $y' \neq y_{max}$  we have

$$s(y_{max}) > s(y')$$

Show

$$\lim_{\beta \rightarrow \infty} P_{\beta \text{ softmax}}(y_{max}) = 1$$