TTIC 31230 Fundamentals of Deep Learning

Problems for GANs.

Problem 1. Conditional GANs In a conditional GAN we model a conditional distribution $\mathrm{Pop}(y|x)$ defined by a population distribution on pairs $\langle x, y\rangle$. For conditional GANs we consider the probability distribution over triples $\langle x, y, i\rangle$ defined by

$$\begin{aligned}
\tilde{P}_\Phi(i=1) &= 1/2 \\
\tilde{P}_\Phi(y|x, i=1) &= \mathrm{pop}(y|x) \\
\tilde{P}_\Phi(y|x, i=-1) &= p_\Phi(y|x)
\end{aligned}$$

(a) Write the conditional GAN adversarial objective function for this problem in terms of $\tilde{P}(x, y, i)$, $P_\Phi(y|x)$ and $P_\Psi(i|y, x)$.

Problem 2. GAN instability

Consider the following adversarial objective where x and y are scalars (real numbers).

$$\max_x \min_y xy$$

(a) Write the differential equation for gradient flow of this adversarial objective.

(b) Give a general solution to your differential equation. (Hint: It goes in a circle). You solution should have parameters allowing for any given initial value of x and y.

Problem 3. Contrastive GANs.

A GAN can be built with a "contrastive" discriminator. Rather than estimate the probability that y is from the population, the discriminator must select which of $y_1, \ldots, y_N$ is from the population.

More formally, for $N \geq 2$ let $\tilde{P}_\Phi^{(N)}$ be the distribution on tuples $\langle i, y_1, \ldots, y_N\rangle$ defined by drawing one "positive" from Pop and $N-1$ IID negatives from P_Φ; then inserting the positive at a random position among the negatives; and returning $(i, y_1, \ldots, y_N)$ where i is the index of the positive.

$$\Phi^* = \operatorname*{argmax}_\Phi \min_\Psi \; E_{(i, y_1, \ldots, y_{N+1}) \sim \tilde{P}_\Phi^{(N)}} \; - \ln p_\Psi(i|y_1, \ldots, y_{N+1}) \quad (1)$$

Restate the above definition of $\tilde{P}^{(N)}_{\Phi}$ and the GAN adversarial objective for the case of conditional constrastive GANs.

Problem 4. Reshaping Noise in GANs. A GAN generator is typically given a random noise vector $z \sim \mathcal{N}(0, I)$. Give equations defining a method for computing z' from z such that the distribution on z' is a mixture of two Gaussians each with a different mean and diagonal covariance matrix. Hint: use a step-function threshold on the first component of z to compute a binary value and use the other components of z to define the Gaussian variables.

Problem 5. This problem is on GAN language modeling. A GAN takes noise as input and transforms it to an output. We consider the case where the output is a string of symbols $w_1, \ldots, w_T$ where for simplicity we always generate a string of exactly length T and where the words are integers with $w_t \in \{0, \ldots, I-1\}$ where I is the size of the vocabulary. The GAN parameters are just the parameters of a bigram model, i.e., the parameters are probability tables

$$
\begin{aligned}
P[i] &= P(w_1 = i) \\
Q[i,j] &= P(w_{t+1} = j \mid w_t = i)
\end{aligned}
$$

We take the noise input to the GAN to be a sequence of random real numbers $\epsilon_1, \ldots, \epsilon_T$ where each ϵ_t is drawn uniformly from the interval $[0, 1]$.

(a) Write a function $\hat{w}(P[I], \epsilon_1)$ which deterministically returns the first word given the noise value ϵ_1 such that the probability over the draw of ϵ_1 that $\hat{w}(P[I], \epsilon_1) = i$ is $P[i]$.

(b) Write a function $\hat{w}(Q[I, I], w_t, \epsilon_t)$ which deterministically returns the word w_{t+1} given w_t such that the probability over the draw of ϵ_t that $\hat{w}(Q[I, I], w_t, \epsilon_t) = j$ is $Q[w_t, j]$.

(c) There is a problem with this GAN. For string generated by the GAN we need to back-propagate the discriminator loss into the GAN generator parameters. Explain why this is problematic. Is this always problematic when the generator output is discrete?